STUDYING MUONIC ATOMS WITH NUMERICAL SOLUTIONS OF THE DIRAC-BREIT-DARWIN EQUATION

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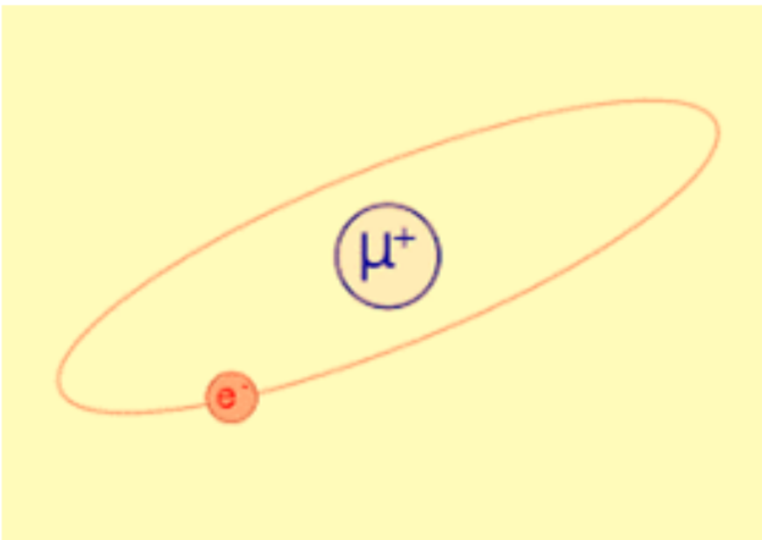
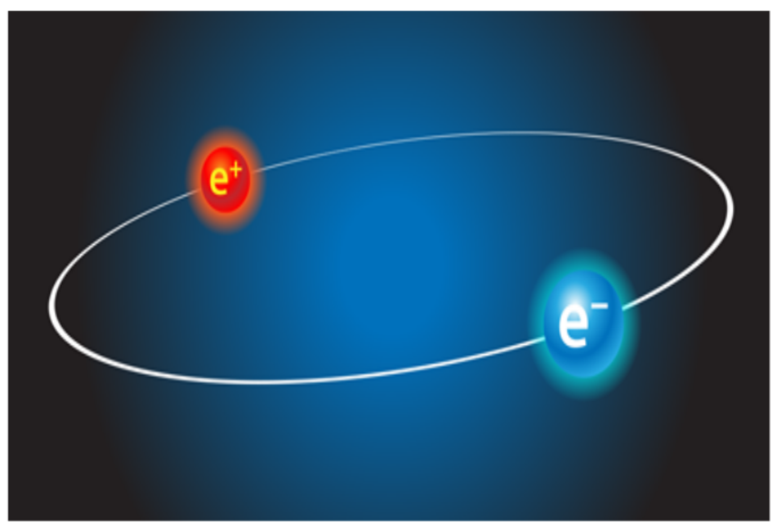
ABSTRACT

It is well known that, the Dirac equation constitutes an optimal approach for the relativistic description of muonic atoms [1,2] as well as purely leptonic atoms like the Muonium (μ^+, e^-) 2-body system [3]. However, several energy corrections like the Breit-Darwin terms, describing additional interactions between the μ^--nucleus or μ^--electron systems, must be included in the Dirac Hamiltonian [4,5]. One of our main goals in this work, is to provide accurate wave functions for the above mentioned systems with numerical solutions of the Dirac equation involving Breit-Darwin terms in order to provide theoretical predictions of high-accuracy for the following: (i) To estimate the contribution of the small (bottom) component, g(r), of the μ^- in muonic atoms. Previous predictions of the ordinary muon-capture rates have either ignored the contribution of g(r) [1] or utilised the non-relativistic Schrödinger equation which by definition consider g(r)=0 [2]. (ii) To calculate the bound spectrum of the Muonium purely leptonic atom that provides excellent tests of the quantum electrodynamics (QED theory) as well as of several beyond the Standard Model (BSM) theories. We mention that, in general the structureless purely leptonic atoms like the Muonium (Mu), Positronium (Ps), etc., are ideal systems for testing the QED and BSM theories. Recently, Mu and Ps are thoroughly being investigated towards the above purpose.

LEPTONIC ATOMS: IDEAL IN TESTING QED AND BSM THEORIES

- Conventional matter consist of leptons and quarks
- Some bosons are responsible for all the interactions
- Leptonic atoms are free of hadronic complex structure and effects ⟶ ideal to investigate the QED and beyond the Standard Model physics
- High precision spectroscopy of Muonium is crucial in testing QED and in determining the m_e/m_μ ratio
- Importance in measuring the muon mass m_μ and fine structure constant α

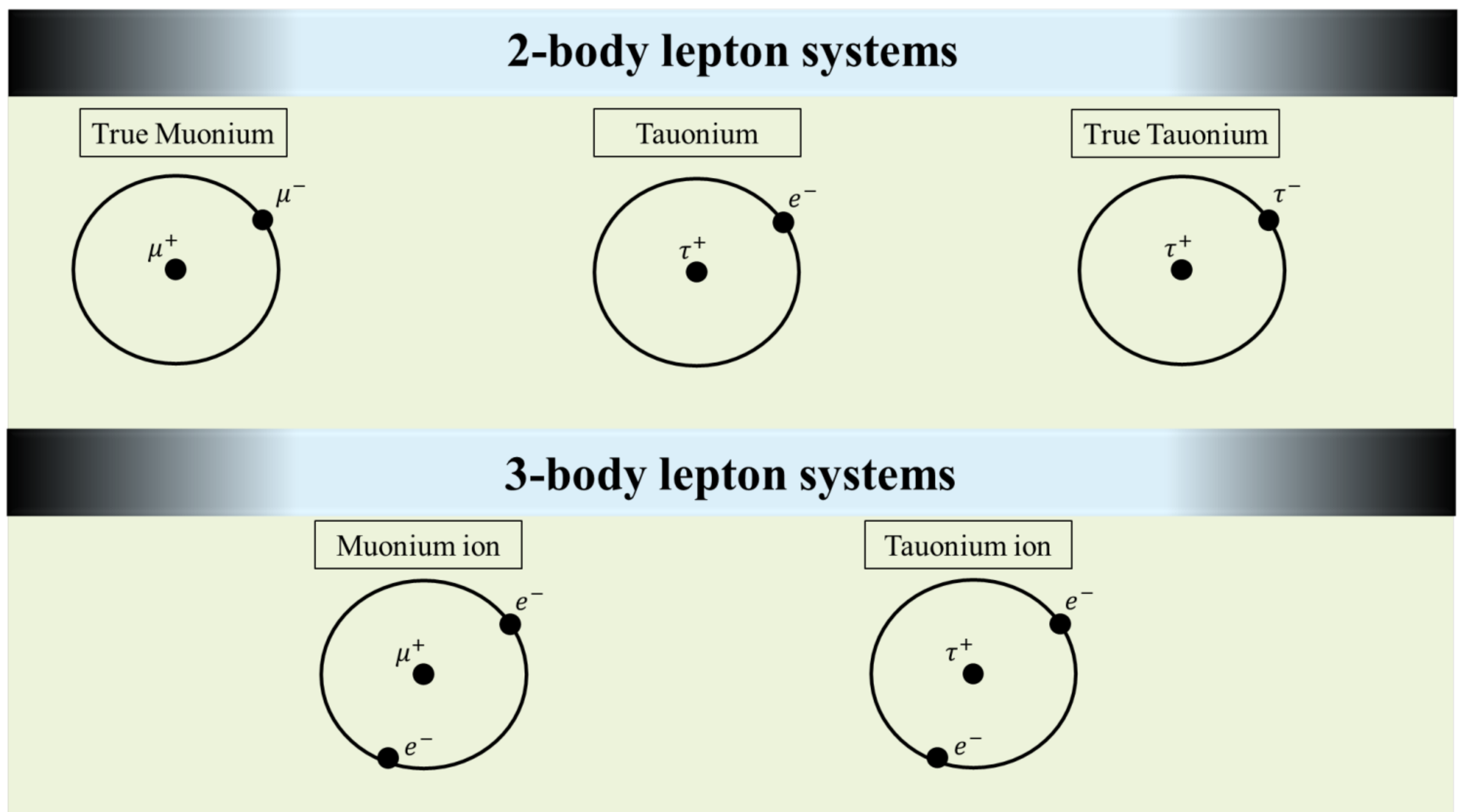




PROPERTIES AND SIGNIFICANCE OF THE MUONIUM (MU) ATOM

- An exotic atom made up from an **antimuon** and an **electron** ($\mu^+ e^-$) where the dominant interaction is electromagnetic
- Due to the large muon's mass m_μ, it shares more similarities to hydrogen than positronium. It may be considered an **exotic light isotope of hydrogen**
- It is **short-lived**, with $\tau = 2,2\ \mu s$, however it undergoes chemical reactions
- μSR is implemented in analysis of the structure of the compounds and chemical transformations
- Muonium is an **ideal system for studying QED and for physics beyond the Standard Model (BSM)**
- **Conversion of *Mu* to $\overline{Mu}$** is effective in identifying fundamental interactions related to the lepton flavor and **lepton number violation**

LEPTONIC ATOMS



QUANTUM MECHANICAL DESCRIPTION

- **Non-relativistic + fine structure**
- **Relativistic - Dirac equation**
- **Breit equation - two-body Dirac equation + Breit interaction**

NON-RELATIVISTIC TREATMENT-THE SCHRÖDINGER EQUATION

Non-relativistic treatment → Schrödinger equation:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2\mu r^2} - \frac{e^2}{r}\right]u(r) = Eu(r)$$

Fine structure splitting → Breaking of the degeneracy of the energy levels due to the spin and relativistic corrections to the Schrödinger equation

I. Relativistic correction to the kinetic energy
II. Spin-orbit coupling
III. Darwin term

→ Derivation from the Non-relativistic limit of the Dirac equation

RELATIVISTIC DESCRIPTION-THE DIRAC EQUATION

$$[\gamma^\mu(i\partial_\mu - eA_\mu) - mc^2]\Psi = 0$$

$$(E - mc^2 - V(r))g\mathcal{Y}_{jlm} = \frac{1}{r}\frac{\boldsymbol{\sigma}\cdot\boldsymbol{r}}{r}\left[-ir\frac{\partial}{\partial r} + i\boldsymbol{\sigma}\cdot\boldsymbol{L}\right]if\mathcal{Y}_{jl'm}$$

$$(E + mc^2 - V(r))if\mathcal{Y}_{jl'm} = \frac{1}{r}\frac{\boldsymbol{\sigma}\cdot\boldsymbol{r}}{r}\left[-ir\frac{\partial}{\partial r} + i\boldsymbol{\sigma}\cdot\boldsymbol{L}\right]g\mathcal{Y}_{jlm}$$

The Dirac equation for a hydrogen-like atom

$$\boldsymbol{\sigma}\cdot\boldsymbol{L}\psi_A = (-k-1)\hbar\psi_A$$
$$\boldsymbol{\sigma}\cdot\boldsymbol{L}\psi_B = (k-1)\hbar\psi_B$$

$$k \to -k-1$$

$$\frac{\partial}{\partial r}\begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} \frac{k-1}{r} & -\frac{E-V-mc^2}{\hbar c} \\ \frac{E-V+mc^2}{\hbar c} & -\frac{k+1}{r} \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}, \quad k = \pm\left(j + \frac{1}{2}\right)$$

$$E_{nj} = \frac{mc^2}{\sqrt{1 + \frac{(Z\alpha)^2}{\left(n - j - \frac{1}{2} + \sqrt{(j+1/2)^2 - (Z\alpha)^2}\right)^2}}}$$

THE BREIT EQUATION

- Semi-relativistic perturbative method - Schrödinger Hamiltonian + Relativistic corrections of Breit equation

$$\hat{\boldsymbol{H}} = \hat{\boldsymbol{H}}_i + \hat{\boldsymbol{H}}_j + \hat{\boldsymbol{H}}_{int}$$

$$\hat{\boldsymbol{H}}_i = \frac{\hat{\boldsymbol{p}}_i^2}{2m_i} - \frac{\hat{\boldsymbol{p}}_i^4}{8c^2m_i^3}$$ → Relativistic correction to the kinetic energy

$$\hat{V}_R = \frac{e^2}{2m_im_jrc^2}\left[\hat{\boldsymbol{p}}_i\cdot\hat{\boldsymbol{p}}_j + \frac{(\boldsymbol{r}\cdot\hat{\boldsymbol{p}}_i)(\boldsymbol{r}\cdot\hat{\boldsymbol{p}}_j)}{r^2}\right]$$ → Retardation effects

$$\hat{V}_D^{(i)} = \frac{\pi e^2\hbar^2}{2m_i^2c^2}\delta(\boldsymbol{r})$$ → Zitterbewegung-Darwin term

$$\hat{V}_{SO}^{(i)} = \frac{-q_ie\hbar^2}{4m_ic^2r^3}\left(\frac{\boldsymbol{r}\times\hat{\boldsymbol{p}}_i}{m_i} - \frac{\boldsymbol{r}\times\hat{\boldsymbol{p}}_j}{m_j}\right)\cdot\boldsymbol{\sigma}_i$$ → Spin-orbit interaction

$$\hat{V}_{hfs} = \frac{e^2\hbar^2}{4m_im_jc^2}\left[\frac{3(\boldsymbol{\sigma}_i\cdot\boldsymbol{r})(\boldsymbol{\sigma}_j\cdot\boldsymbol{r}) - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j}{r^3} + \frac{8\pi}{3}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\delta(\boldsymbol{r})\right]$$ → Hyperfine structure hamiltonian

$$\hat{V}_{ext}^{(i)} = \frac{e\hbar}{m_ic}\left(\frac{\boldsymbol{H}\cdot\boldsymbol{\sigma}_i}{2} + \frac{q_i}{m_ic}\boldsymbol{A}\cdot\hat{\boldsymbol{p}}_i\right)$$ → Interaction with external magnetic field

- Breit interaction as a perturbation in the two-body Dirac hamiltonian - Dirac-Coulomb-Breit Hamiltonian

$$\left[c\boldsymbol{\alpha_1}\cdot\boldsymbol{p_1} + \beta_1m_1c^2 + c\boldsymbol{\alpha_2}\cdot\boldsymbol{p_2} + \beta_2m_2c^2 + V_C + B\right]\Psi = E\Psi, \quad B = -\frac{q_1q_2}{2}\left[\frac{\boldsymbol{\alpha}_1\cdot\boldsymbol{\alpha}_2}{r} + \frac{(\boldsymbol{\alpha}_1\cdot\boldsymbol{r})(\boldsymbol{\alpha}_1\cdot\boldsymbol{r})}{r^3}\right]$$

MUONIUM EXPERIMENTAL TARGETS

Mu spectroscopy

- Ground state hfs structure
- 1s-2s energy interval
- *Mu-$\overline{Mu}$* conversion
- Test of CPT and Lorentz invariance

Achievements

- Accuracy in values such as m_μ, μ_μ, α_μ, α
- Search for new unknown forces in nature
- Restrictions on parameters of speculative theories

LOOKING FORWARD TO...

Important upcoming Mu experiments

- MuSEUM at J-PARC → the hfs structure
- Mu-MASS at PSI → 1S-2S transition frequency
- Muon g-2 at FNAL → anomalous magnetic moment

CONCLUSIONS

- **Purely leptonic atoms offer promising research prospects in testing QED and BSM theories**
- **Mu is leading in the research interest due to its unique characteristics**
- **Accurate theoretical predictions require the solution of the Dirac equation, taking into account the Breit-Darwin terms**
- **Promising new experiments aim at shedding light on theoretical contradictions, especially concerning Mu**
- **Our goal is to make improved theoretical predictions for the leptonic atom bound states using advanced numerical codes**

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