Open broblems in conventional & exotic muon physics: Predictions through numerical solutions of fundamental differential equations

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Aim: "Modelling and Development of Advanced Numerical Tools to Solve the Fundamental Dirac-Breit-Darwin Differential Equations: Algorithms Assessment on conventional & exotic processes taking place in: (i) The muonic atoms, and (ii) The Exotic purely, Leptonic Atoms, Promising Probes to Test the QED and BSM Theories"

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Overview

1. Introduction
   - Motivation and Research Challenges
   - Objectives of the Project
2. State-of-the-art of the research field
   - Experiments studying processes in muonic-atoms :
     - Conventional μ^--capture on nuclei
     - Exotic μ^--processes in the presence of nuclei $(\mu^- \rightarrow e^-,\ \mu^- \rightarrow e^+)$
   - Experiments: Muon Spectroscopy
   - Theory of exotic pure leptonic atoms
3. Mathematical and Computational Modelling
   - The Dirac-Breit-Darwin Differential Equation in leptonic atoms
   - Computational Methodology in Neural Network Techniques
4. Summary, Future Steps of the Project

Conventional and exotic matter (atoms) in our world

- According to theory of atoms, all matter of our world is made of conventional atoms: p^+, n^0, e^-.
- Current extremly sensitive detectors and accelerators discovered exotic forms of matter, exotic leptonic atoms (not made of the 3 basic constituents, but only of leptons).
- Leptonic atoms do not have structure (finite-size effects), making them ideal for
  - fundamental physical constants: muon mass, muon magnetic moment (g-2 exp. FNAL)
  - testing bound-state QED and searching for BSM physics

The simplest and some of the most extensively studied bound atomic systems are

- The hydrogen atom, $\mathrm{H} = (p^+, e^-)$, The muonic-hydrogen, $\mu^+\mathrm{H} = (p^+, \mu^-)$
- The positronium, $\mathrm{Ps} = (e^+, e^-)$, The Muonium, $\mathrm{Mu} = (\mu^+, e^-)$

Their simplicity allows "high-precision evaluation of energy corrections" as:

- (i) Relativistic corrections, (ii) Corrections coming from matter-photon interactions

H-atoms used to test QED, however, the proton-finite-size reduces the testing accuracy in H and $\mu^+ H$. The structureless *Mu* and *Ps*, arose theoretical and experimental interest, recently

Motivation, Research Challenges, and Project's Objectives

Challenge 1: The description of structure and evolution of Quantum Systems, requires accurate theoretical predictions relying on W-Fs, accurate solutions of Diff. Eqs. (Schrödinger, Dirac, etc.)

Challenge 2: Recently, purely leptonic atoms have triggered intense interest of researches. Precision spectroscopy using low-energy muon-beams at PSI, J-PARC, Fermilab, RCNP, etc., are performed.

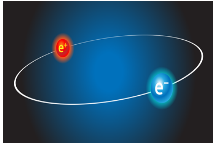
Challenge 3: The structureless (μ^+, e^-), Mu, and (e^+, e^-), Ps, systems are qualified as probes to study QED & BSM physics. Experimentally, Mu and Ps are considered promising to do precise spectroscopy.

Objective 1: To model and build advanced numerical codes (in Python) for solving accurately differential Eqs. as the Dirac-Breit-Darwin required for the description of Mu, Ps, Mu^-.

Objective 2: To exploit the high precision measurements for Mu, Ps, and Mu^-, to assess the efficiency of our algorithms in testing QED and BSM theories.

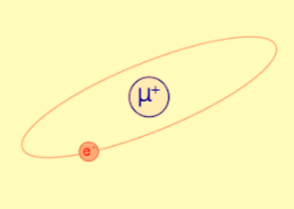
Objective 3: To test some symmetries in physics, and some particle physics scenarios by constraining parameters entering specific BSM interaction mechanisms

Examples of 2-body purely leptonic atoms



The Positronium (Ps)

- Theoretically, several leptonic atoms can be studied
- Not all combinations of $e^{\pm}, \mu^{\pm}, \tau^{\pm}$ result to bound systems, exotic leptonic atoms
- Up to now, only few exotic atoms (Mu, Ps, M^{-} ...) have been detected and studied both experimentally and theoretically



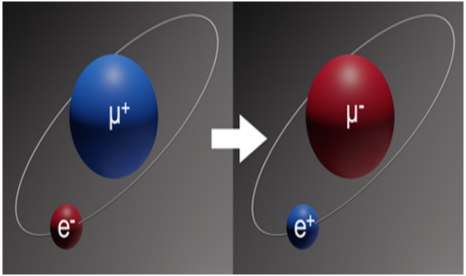
Exotic Mu atom

- In this talk, we discuss exotic systems, few body systems: $e^{\pm}, \mu^{\pm}, \tau^{\pm}$, in particular:
  - their bound state W-Fs and probability densities
  - the low-lying energy spectra
- In general, with our method we may study systems of two- or three-body like, (μ^{+}, τ^{-}), (μ^{+}, μ^{-}), (e^{+}, e^{-}), $(\mu^{+}, e^{-}\tau^{-})$, etc.
- The investigation of the stability of exotic systems with muons and/or tauons is of fundamental interest in (QED).

Studying BSM physics with exotic leptonic atoms

- One of the best purely leptonic systems, where BSM physics could be studied is the *Mu* to anti-*Mu* Conversion
- The latter oscillation-process has been recently studied theoretically and experimentally

- Recent calculations employed non-relativistic Schrödinger Eq. for Mu and anti-Mu.
- In this project, we are able to compare Dirac-Coulomb-Breit Eq. results with the previous ones.



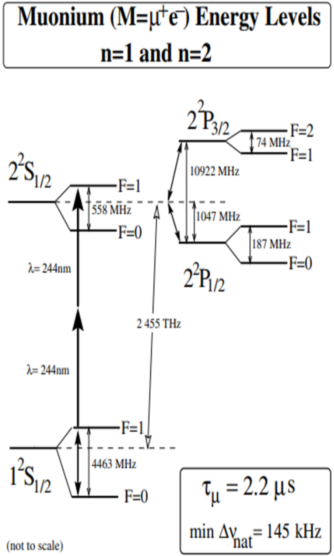
Uesaka, NEW-Colloquium, 2023, RCNP Osaka, Japan. See PRD work of his group

Mu Spectrum for $[n = 1, \ell = 0]$ and $[n = 2, \ell = 0, 1]$

- Experiments provide the bound energy-spectra of Mu
- The Mu-MASS experiment (PSI) aims to measure the 1S-2S transition of Mu to an unprecedented precision of 10 kHz, a 1000-fold improvement over previous measurements!

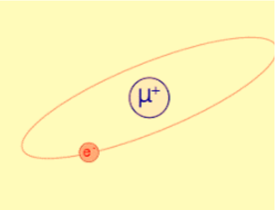
MU-MASS

- We plan to asses our method in predicting this transition

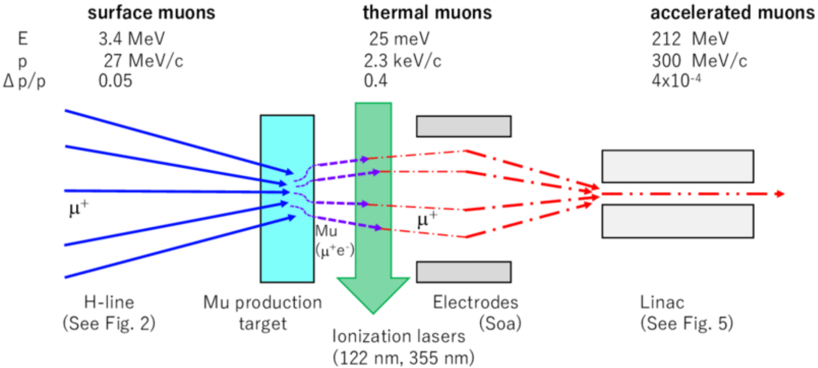


Experimental Study of Muonium

Experimentally, the Mu is produced when an anti-muon (μ^+) beam hits a target of Al, etc.



Exotic Mu atom



The outgoing beam contains: Mu, μ^+, and $(\mu^+, e^- e^-)$ called Mu ion

The Muonium exotic atom

- Mu Spectroscopy provides a rigorous validation of QED through a comparison with theoretical predictions.
- At J-PARC MLF MUSE, a series of microwave spectroscopy experiments of the ground-state hyperfine splitting in Mu is in progress.
- Low-energy muons obtained at MUSE through the laser ionization of Mu can be employed to use slow Mu with minimal temporal and spatial spread.
- A Mu interferometer possesses various potential applications as
  - a measurement of the Berry phase, and
  - a precise measurement of the muon mass
- The MuSEUM collaboration performed a new precision measurement of Mu ground-state hyperfine-structure at J-PARC using:
  - "a high-intensity pulsed muon beam
  - "a high-rate capable positron counter"

The Dirac Equation Formalism

In external electromagnetic 4-vector potential, the Dirac Eq. takes the form:

$$[\gamma^\mu(i\partial_\mu - eA_\mu) - mc^2]\Psi = 0\,,$$

known as minimal coupling Dirac Eq. (due to the substitution $\quad p_\mu \to p_\mu - eA_\mu$)

By assuming that $\mathbf{A} = 0$ and that the large and small components of Ψ are

$$\psi_A = g(r)\mathcal{Y}^{m_j}_{jl_A}, \qquad \psi_B = if(r)\mathcal{Y}^{m_j}_{jl_B}$$

[$\mathcal{Y}^{m_j}_{jl_B} \longrightarrow$ spin-spherical harmonic] Dirac Eq. takes the form

$$(E - mc^2 - V(r))f\mathcal{Y}_{jlm} = \frac{1}{r}\frac{\sigma\cdot\mathbf{r}}{r}\left[-ir\frac{\partial}{\partial r} + i\sigma\cdot\mathbf{L}\right] ig\mathcal{Y}_{jl'm}$$

$$(E + mc^2 - V(r))ig\mathcal{Y}_{jl'm} = \frac{1}{r}\frac{\sigma\cdot\mathbf{r}}{r}\left[-ir\frac{\partial}{\partial r} + i\sigma\cdot\mathbf{L}\right] f\mathcal{Y}_{jlm}$$

The radial f(r) and g(r) Dirac-Coulomb wavefunctions

Finally, the radial wave functions: f(r) and g(r), satisfy the two coupled differential Eqs.

$$\frac{\partial}{\partial r}\begin{bmatrix} f(r) \\ g(r) \end{bmatrix} = \begin{bmatrix} \frac{k-1}{r} & -\frac{E-V-\mu c^2}{\hbar c} \\ \frac{E-V+\mu c^2}{\hbar c} & -\frac{k+1}{r} \end{bmatrix} \begin{pmatrix} f(r) \\ g(r) \end{pmatrix}$$

where $k = \pm\left(j + \frac{1}{2}\right)$
The energy eigenvalues are given by

$$E_{nj} = mc^2 \left[1 + \frac{\alpha^2}{\left[n - j - \frac{1}{2} + \sqrt{(j+1/2)^2 - \alpha^2}\right]^2}\right]^{-1/2}$$

Note: Dirac Eq (unlike Klein-Gordon Eq.) admits a conserved 4-current with a non-negative definite time component. So, this current can be interpreted as a probability 4-current.

Kena, E. D. and Adera, G. B. (2021) J. Phys. Commun. 5 105018

The Dirac-Coulomb-Breit Equation

The Dirac-Coulomb equation that includes the Breit term provides more complete relativistic description for the leptonic atoms. This is written as

$$i\hbar\frac{\partial\Psi}{\partial t} = \left(\sum_i \hat{H}_D(i) + \sum_{i>j}\frac{1}{r_{ij}} + \sum_{i>j}\hat{B}_{ij}\right)\Psi$$

where the first term $\hat{H}_D(i)$ is the Dirac Hamiltonian, while

$$\hat{B}_{ij} = -\frac{1}{2r_{ij}^3}\left[a(i)\cdot a(j)r_{ij}^2 + (a(i)\cdot r_{ij})(a(j)\cdot r_{ij})\right]$$

represent the Breit term. The Dirac-Coulomb-Breit Hamiltonian includes the terms:

$$\hat{\mathbf{H}}_i = \frac{\hat{\mathbf{p}}_i^2}{2m_i} - \frac{\hat{\mathbf{p}}_i^4}{8c^2m_i^3}$$

which is related to the relativistic velocity of each lepton

The Dirac-Coulomb-Breit Equation

$$\hat{V}_R = \frac{e^2}{2m_i m_j r c^2}\left[\hat{\mathbf{p}}_i \cdot \hat{\mathbf{p}}_j + \frac{(\mathbf{r}\cdot\hat{\mathbf{p}}_i)(\mathbf{r}\cdot\hat{\mathbf{p}}_j)}{r^2}\right]$$

$$\hat{V}_{Dar}^{(i)} = \frac{\pi e^2 \hbar^2}{2m_i^2 c^2}\delta(\mathbf{r})$$

The latter is known as the Darwin term

$$\hat{V}_{SO}^{(i)} = \frac{-q_i e\hbar^2}{4m_i c^2 r^3}\left(\frac{\mathbf{r}\times\hat{\mathbf{p}}_i}{m_i} - \frac{\mathbf{r}\times\hat{\mathbf{p}}_j}{m_j}\right)\cdot\boldsymbol{\sigma}_i$$

$$\hat{V}_{hfs} = -\frac{e^2\hbar^2}{4m_i m_j c^2}\left[\frac{\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j - 3(\boldsymbol{\sigma}_i\cdot\mathbf{r})(\boldsymbol{\sigma}_j\cdot\mathbf{r})}{r^3} - \frac{8\pi}{3}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\delta(\mathbf{r})\right]$$

$$\hat{V}_{ext}^{(i)} = \frac{e\hbar}{m_i c}\left(\frac{\mathbf{H}_{ext}\cdot\boldsymbol{\sigma}_i}{2} + \frac{q_i}{m_i c}\mathbf{A}_{ext}\cdot\hat{\mathbf{p}}_i\right)$$

Note that, the Breit-Darwin Eq. is not invariant under Lorentz transformation.

Daza, F.G., Kelkar, N.G. and Nowakowski, M. (2012) J. Phys. G: Nucl. Part. Phys. 39 035103

Solving the 2-body Dirac Eq. with neural-networks

- Physically, the Dirac Eq. describes relativistic systems consisting of spin-1/2 particles in atomic, nuclear, and particle physics.
- Mathematically, the Dirac Eq. is a coupled first-order differential equation for the large f(r) and small g(r) components.
- Analytically, Dirac Eq. can be solved with few potentials so, **numerical methods are highly demanded** to obtain the eigenenergies and wave functions in the relevant systems.
- Computationally, many numerical schemes have been applied to solve the Dirac Eq.
- The goal of our Team, is to solve the Dirac (+Breit+Darwin terms) Eq. for relativistic pure leptonic atoms, within neural networks.
- Our advanced numerical method, achieves great success in solving Schrödinger Eq.
- Currently, the method is applied to solve the **Dirac-Coulomb Eq.** for 2-body exotic leptonic atoms Mu, Ps, true Mu, etc.

Summary, Future Steps of the Project

- For the description of exotic leptonic atoms, Ps, Mu, etc., one starts by solving the 2-body Schrödinger Eq.
  - The (nonrelativistic) Schrödinger Eq. has analytic solutions for two-lepton systems.
  - In the literature, Schrödinger Eq. solutions are used as reference
  - We also started from the Schrödinger Eq. and solved it numerically (code, in Python, is based on N.N.) for the Mu and Ps systems
- The 2-body Dirac–Coulomb Eq. has analytic solutions too. Our N.N. code, in Python, is ready to provide the corresponding numerical solutions for exotic leptonic atoms
- In the next step, the 2-body Dirac-Coulomb-Breit Eq. will be numerically solved
- Finally, the 3-body Dirac-Coulomb-Breit Eq. is planned to be solved for the Muonium ion (Mu^-) system
- In the last two cases BSM Physics could be accurately studied. BSM-Model parameters would be constrained

Collaborators: OPRA-UoI-UoJ Project's Research Team

- Theocharis Kosmas, Univ. of Ioannina, Greece: S.C. of OPRA-UoI-UoJ Project
- Jouni Suhonen, Univ. of Jyvaskyla, Finland: Leader of testing QED and BSM theories
- Odyssefs Kosmas (Principal Investigator), ConIgital, Manchester, Birmingham-Coventry, UK: Leading the derivation and assessment of the new algorithms
- Angellos Giotis (PostDoc), derivation of algorithms (in Python) with Deep Neural Networks
- Athanasios Gkrepis (PhD St), derivation and assessment of the new algorithms (in Python)
- Theodora Papavasileiou (PhD St), testing QED theory with the new algorithms
- Georgios Gkartzonikas (PhD St), derivation of the Simulated Annealing algorithm (in Python language)
- Leandros Perivolaropoulos (Deputy Scientific Coordinator), Univ. of Ioannina, Greece

T.S. Kosmas is co-advisor of PhD Studs.: Th. Papavasileiou and A. Gkrepis

Thank you for your attention !

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